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Suppression of Gamow-Teller and M1 transitions in deformed mirror nuclei ^{25}Mg and ^{25}Al

Direct observation of K selection rules

Y. Shimbara[1], Y. Fujita[1,a], T. Adachi[1], G.P.A. Berg[2,3], H. Fujita[1,b], K. Fujita[2], I. Hamamoto[4], K. Hatanaka[2], J. Kamiya[2,c], K. Nakanishi[2], Y. Sakemi[2], Y. Shimizu[2], M. Uchida[5], T. Wakasa[2,d], and M. Yosoi[5]

[1] Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
[2] Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
[3] Kernfysisch Versneller Instituut, Zernikelaan 25, 9747 AA Groningen, The Netherlands
[4] Division of Mathematical Physics, LTH, University of Lund, P.O. Box 118, S-22100 Lund, Sweden
[5] Department of Physics, Kyoto University, Sakyo, Kyoto 606-8502, Japan

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Abstract. The mirror nuclei ^{25}Mg and ^{25}Al are expected to have very similar structures. The Gamow-Teller (GT) transitions from the $J^\pi = 5/2^+$ ground state of ^{25}Mg to the excited states in ^{25}Al were studied by high-resolution measurements of the ^{25}Mg(^{3}He, t) charge-exchange reaction at 0° and at 140 MeV/nucleon. Assuming the usual $\Delta J^\pi = 1^+$ selection rule for the spin-isospin–type GT transitions, the states with $J^\pi = 3/2^+, 5/2^+$, and $7/2^+$ should be excited. However, of the more than ten states with these J^π values below 6 MeV excitation energy, only the $5/2^+$ ground state and the $7/2^+$, 1.613 MeV state in ^{25}Al were strongly populated, while all other states were strongly suppressed. The analysis of $M1$ transitions in ^{25}Mg also suggested a very similar feature for the analogous $M1$ transitions. Both ^{25}Mg and ^{25}Al are known to be largely deformed, and most low-lying states can be interpreted in terms of one-particle quantum numbers in the deformed potential and the associated rotational spectra. The observed suppression can be explained in terms of the K quantum number selection rules that are inherent to axially deformed nuclei.

PACS. 21.10.Re Collective levels – 21.60.Ev Collective models – 25.55.Kr Charge-exchange reactions – 27.30.+t $20 \leq A \leq 38$

1 Introduction

The Gamow-Teller (GT) excitation caused by the $\sigma\tau$ operator is the simplest spin excitation without angular-momentum transfer ($\Delta L = 0$), and, therefore, is associated with the $\Delta J^\pi = 1^+$ selection rule, where J^π denote the total spin and the parity. The GT states are selectively excited in β-decays as well as in charge-exchange (CE) reactions at 0° and at intermediate incident energies [1,2].

In a deformed nucleus with z-axis symmetry, the z component K of the total spin J is a good quantum number. Each single nucleon is in a Nilsson orbit labeled by the asymptotic quantum numbers $[Nn_z\Lambda\Omega]$ [3], where N is the total oscillator quantum number, n_z the number of quanta along the z-axis, and Λ and Ω are z-axis projections of the orbital and total angular momenta. Low-lying states of an odd-mass deformed nucleus are well described in terms of the particle-rotor model assuming both a single quasi-particle in various Nilsson orbits as the intrinsic configuration and the collective rotation induced by the core. Since the rotation of the core is perpendicular to the z-axis, $K = \Omega$ holds. Therefore, each rotational band is specified by the quantum numbers of the single-particle orbit $K^\pi[Nn_z\Lambda]$.

In the middle of the sd shell, nuclei with mass number $19 \leq A \leq 25$ are strongly deformed [3]. The static quadrupole moment Q_{2^+} of the first 2^+ state of the even-even nucleus ^{24}Mg is about -18 fm^2 [4]. This large value suggests that ^{24}Mg has a prolate deformation with a deformation parameter $\delta \approx 0.4$–0.5. Low-lying states in the $A = 23$, $T_z = \pm 1/2$ mirror nuclei ^{23}Na and ^{23}Mg and those in the $A = 25$ system ^{25}Mg and ^{25}Al are well described in terms of the particle-rotor model [3,5–7], where

[a] e-mail: fujita@rcnp.osaka-u.ac.jp
[b] *Present address*: Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan.
[c] *Present address*: Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195, Japan.
[d] *Present address*: Department of Physics, Kyushu University, Higashi, Fukuoka 812-8581, Japan.

T_z is the z component of isospin T. The GT excitations in the $A = 23$ system were studied previously in the ^{23}Na(^{3}He, t)^{23}Mg reaction at 0° [8]. With the high resolution of the measurement, many prominent peaks of GT excitations were observed, as shown in fig. 1a). The obtained GT strengths were compared with the strengths of the analogous $M1$ transitions.

The study is extended to the $A = 25$ system by the same reaction on a ^{25}Mg target. Although the mass number difference of these two systems is only two, we found that the measured ^{25}Al spectrum, selectively showing GT excitations, is completely different at excitation energies (E_x) below 6 MeV. In the $A = 25$ system most of the GT excitations are very much suppressed except for the transitions to the ground and 1.613 MeV states. We also found a suppression of analogous $M1$ transitions in the corresponding energy region of ^{25}Mg from the analysis of γ-decay data [9] and data of electron inelastic scattering (e, e') [10].

Both $M1$ and GT operators have the same major isovector (IV) spin ($\sigma\tau$) component. The electromagnetic $M1$ operator contains additionally IV orbital ($\ell\tau$), isoscalar (IS) spin (σ), and IS orbital (ℓ) terms [11,12]. The orbital contributions for $M1$ transitions can be very large in deformed nuclei, as was shown for the $A = 23$ mirror nuclei ^{23}Na and ^{23}Mg [8].

In this paper, we will explain the suppression of GT and $M1$ transitions in the $A = 25$ system on the bases of the selection rules of the K quantum number inherent to rotational bands in addition to the usual J^π selection rule inherent to each state. We will also present results of the orbital contributions in $M1$ transitions.

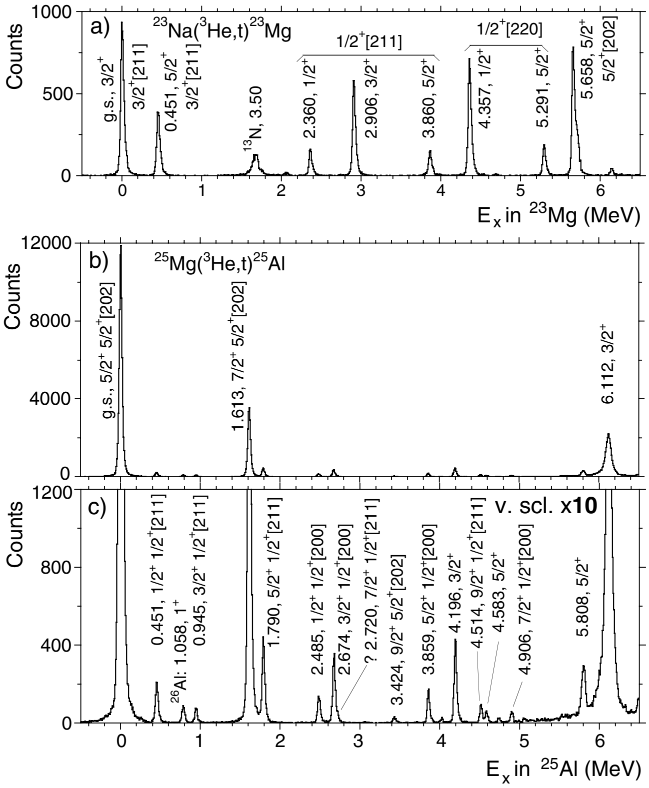

Fig. 1. Comparison of a) ^{23}Na(^{3}He, t)^{23}Mg spectrum and b) ^{25}Mg(^{3}He, t)^{25}Al spectrum. The ordinates of figs. a) and b) are scaled so that states with similar B(GT) values have similar peak heights. The spectrum shown in b) is repeated in c) with the vertical scale expanded by a factor of ten (v. scl. ×10) in order to show weakly excited states more clearly. Excitation energy (in units of MeV), J^π, and the rotational band are indicated for each state.

2 Experiment

According to the $\Delta J^\pi = 1^+$ selection rule, GT transitions are allowed from the ground state of ^{25}Mg with $J^\pi = 5/2^+$ to the $J^\pi = 3/2^+$, $5/2^+$, and $7/2^+$ "GT states" in ^{25}Al. The compilation by Endt [9] shows that more than ten states with one of these J^π values are in the $E_x < 6$ MeV region. We studied the transition strengths to these states by using the (^{3}He, t) reaction at 0°.

At intermediate energies (≥ 100 MeV/nucleon) and at forward angles including 0°, GT states become prominent in CE reactions, like (^{3}He, t), because of their $L = 0$ nature and the dominance of the $\sigma\tau$ part of the effective nuclear interaction [2,13]. A ^{25}Mg(^{3}He, t)^{25}Al experiment was performed at RCNP, Osaka by using a 140 MeV/nucleon ^{3}He beam from the $K = 400$, RCNP Ring Cyclotron and the Grand Raiden spectrometer [14]. The large difference of energy losses of ^{3}He and tritons in a target foil can cause a large energy spread of the outgoing tritons and deteriorate the spectral resolution. Therefore, a thin self-supporting ^{25}Mg target with a thickness of 0.93 mg/cm^2 was used. The isotopic enrichment of ^{25}Mg was 98.3%.

The outgoing tritons were momentum-analyzed within the effective full acceptance of the spectrometer (horizontally ≈ ± 20 mrad and vertically ≈ ± 40 mrad). The focal-plane detector system consisted of two sets of multi-wire drift chambers [15] followed by two plastic scintillation detectors. The scintillation detectors provided a fast timing signal for time-of-flight information and also energy loss signals for particle identification. The wire chambers allowed track reconstructions in horizontal and vertical directions for each ray. The acceptance of the spectrometer was subdivided in the software analysis by using the track information.

An energy resolution far better than the energy spread of the beam was realized by applying *dispersion matching* and *focus matching* techniques [16]. In order to realize these matching conditions, the new high-resolution "WS course" [17] for the beam transportation and the "faint beam method" [18,19] to diagnose the matching conditions were utilized. In the present measurement, a very good energy resolution of 35 keV (full width at half-maximum (FWHM)) was achieved. With this improved resolution, states up to $E_x = 6$ MeV were clearly resolved, as shown in figs. 1b) and c). Many of these states are so weak that they can be seen only in fig. 1c) with an expanded vertical scale. The 2.720 MeV, $7/2^+$ state can be barely seen at the right shoulder of the 2.674 MeV peak.

Table 1. Low-lying states in ^{25}Al and ^{25}Mg with $J^\pi = 3/2^+$, $5/2^+$, and $7/2^+$. The GT transition strengths $B(\text{GT})$ from the $^{25}\text{Mg}(^{3}\text{He}, t)^{25}\text{Al}$ reaction and the $B(M1)$ strengths of the analogous $M1$ transitions in ^{25}Mg are listed. The ratios R_{ISO} calculated from these $B(M1)$ and $B(\text{GT})$ values are given, where $R_{\text{MEC}} = 1.25$ is assumed. Excitation energies are in units of MeV, and the $B(M1)$ values are in units of μ_N^2. Errors of excitation energies are shown only where $\Delta E > 1$ keV.

States in ^{25}Al			States in ^{25}Mg						
		(^{3}He, t)		γ-decay			(e, e')		
$E_x^{(\text{a})}$	$2 \cdot J^{\pi\,(\text{b})}$	$B(\text{GT})^{(\text{c})}$	$E_x^{(\text{a})}$	$B(M1)^{(\text{a})}$	$B^R(M1)$	R_{ISO}	$B(M1)^{(\text{d})}$	$B^R(M1)$	R_{ISO}
0.0	5^+	$0.408(2)^{(\text{e})}$	0.0						
0.945	3^+	$0.003(1)^{(\text{f})}$	0.975	0.0011(1)					
1.613	7^+	$0.165(7)^{(\text{e,g})}$	1.612	0.83(12)	0.63(9)	3.1(8)	1.2(3)	0.87(22)	4.2(11)
1.790	5^+	$0.019(2)^{(\text{f})}$	1.965	0.0014(2)					
2.674	3^+	$0.017(2)^{(\text{f})}$	2.801	0.007(2)					
2.720	7^+		2.738						
3.859	5^+	$0.007(1)^{(\text{f})}$	3.908						
4.196(3)	3^+	$0.019(2)^{(\text{f})}$	4.359						
4.583(4)	5^+	$0.002(1)^{(\text{f})}$	4.722						
4.906(4)	7^+		5.012						
5.808(6)	5^+	$0.020(4)^{(\text{f})}$							
6.122(3)	3^+	0.235(29)	5.747	0.11(5)	0.08(4)	0.33(17)	0.27(13)	0.20(10)	0.8(4)

(a) From ref. [9].
(b) From refs. [3,9].
(c) Present work.
(d) From ref. [10].
(e) $B(\text{GT})$ value from β-decay measurement.
(f) The obtained $B(\text{GT})$ value is small and less reliable, see text.
(g) $B(\text{GT})$ value used as a standard.

It is instructive to compare this "$A = 25$" spectrum with the "$A = 23$" spectrum shown in fig. 1a). The scales of ordinates in figs. 1a) and b) are adjusted so that GT excitations with the same $B(\text{GT})$ values show approximately the same peak heights. The suppression of the GT excitations in ^{25}Al compared to those in ^{23}Mg is very obvious.

In order to accurately determine the scattering angle Θ near 0°, angle measurements in both the x-direction (θ) and y-direction (ϕ) are equally important, where Θ is defined by $\Theta = \sqrt{\theta^2 + \phi^2}$. Good θ and ϕ resolutions were achieved by applying the *angular dispersion matching* technique [16] and the "overfocus mode" of the spectrometer [20], respectively. It is estimated that a scattering-angle resolution of 6–8 mrad (FWHM) was achieved. The "0° spectra" in figs. 1b) and c) show events for scattering angles $\Theta \leq 0.8°$. The excitation energies of ^{25}Al given in figs. 1b) and c), and also in table 1 are from ref. [9]. They were in agreement within an error of 5 keV with our experimental values (for details of the energy calibration, see ref. [8]). For the states below 4 MeV, the errors of excitation energies given in ref. [9] are smaller than 1 keV. The J^π values of ^{25}Al given in table 1 are also from ref. [9]. For the 1.613 MeV and 4.906 MeV states, values are given assuming that the J^π values of the isobaric analog states in ^{25}Mg and ^{25}Al listed in refs. [3,9] should be the same.

The counts of individual peaks were obtained by applying a peak-fitting program using the shape of the well-separated $J^\pi = 7/2^+$ GT state at 1.613 MeV as standard. Obvious broadening is seen only for the 6.112 MeV peak, although the proton separation energy S_p is 2.27 MeV. In order to examine the $L = 0$ nature of states, the yield ratio of each state in the spectra with angle cuts $\Theta = 1.5°$–$2.0°$ and $\Theta = 0°$–$0.5°$ was derived and compared with the ratio of the 1.613 MeV state, the strongly excited GT state expected to represent the $L = 0$ angular distribution. It was found that the yield ratios for states associated with $L > 0$ transfer ($\Delta J^\pi \neq 1^+$) were larger by more than 20% compared to the ratio of the 1.613 MeV state. On the other hand, all observed $\Delta J^\pi = 1^+$ states, even very weakly excited states, had deviations less than 15%, suggesting that they have angular distributions inherent to $L = 0$ transfer in the vicinity of $\Theta = 0°$. These $\Delta J^\pi = 1^+$ GT states are listed in table 1 by their excitation energies together with their J^π values.

3 Data analysis

3.1 B(GT) evaluation from (^{3}He, t) data

It is known that the cross-sections for GT transitions are approximately proportional to $B(\text{GT})$ values in CE reactions at 0° and at intermediate incident energies [21–23]:

$$\frac{\mathrm{d}\sigma_{\text{CE}}}{\mathrm{d}\Omega}(0°) \simeq K\, N_{\sigma\tau}\, |J_{\sigma\tau}(0)|^2 B(\text{GT}), \tag{1}$$

where $J_{\sigma\tau}(0)$ is the volume integral of the effective interaction $V_{\sigma\tau}$ at momentum transfer $q = 0$, K is the kinematic factor, and $N_{\sigma\tau}$ is a distortion factor. A study of analogous GT transitions in $T = 1/2$, $A = 27$ mirror nuclei ^{27}Al and ^{27}Si [24], and a study of $T_z = \pm 1 \rightarrow 0$ GT transitions in $A = 26$ nuclei ^{26}Mg, ^{26}Al, and ^{26}Si [25] showed that the proportionality is valid for the transitions with $B(\mathrm{GT}) \geq 0.04$ in $(^3\mathrm{He}, t)$ reactions.

In order to obtain $B(\mathrm{GT})$ values by using eq. (1), a standard $B(\mathrm{GT})$ value is needed. We used the $B(\mathrm{GT})$ value of 0.165 ± 0.007 obtained in the β-decay from the ^{25}Al ground state to the 1.612 MeV state of ^{25}Mg (see table 1 and ref. [9]). If isospin symmetry of mirror nuclei is assumed, it is expected that the $B(\mathrm{GT})$ values of mirror transitions are the same. We postulated that the transition to the 1.613 MeV state in ^{25}Al has this $B(\mathrm{GT})$ value in the $^{25}\mathrm{Mg}(^3\mathrm{He}, t)$ reaction. The $B(\mathrm{GT})$ values for other excited GT states can be calculated by using the proportionality from their peak counts at 0°. The product $KN_{\sigma\tau}$ in eq. (1) changes gradually as a function of excitation energy [22]. In order to estimate this effect, a distorted-wave Born approximation (DWBA) calculation was performed by using the code DW81 [26], assuming a simple $d_{5/2} \rightarrow d_{3/2}$ transition for the excited GT states (for details, see ref. [8]). The resulting $B(\mathrm{GT})$ values including this correction of up to 6% are listed in table 1. Since both $\sigma\tau$ and τ operators contribute to the transition between ground states, a separate extraction of the GT strength is not possible from the present $(^3\mathrm{He}, t)$ measurement. Therefore, the $B(\mathrm{GT})$ value from the mirror symmetry β-decay is given for the ground state. As we see, except for the ground, 1.613 MeV, and 6.122 MeV states, the $B(\mathrm{GT})$ values are very small. As mentioned, the $B(\mathrm{GT})$ values smaller than 0.04 may be less reliable. However, the important fact is that small $B(\mathrm{GT})$ values in the β-decay are also small in the present CE reaction [24].

3.2 B(M1) strengths

Very similar structures have been proposed for mirror nuclei ^{25}Mg and ^{25}Al [3], as expected from isospin symmetry. The strengths of the $M1$ transitions in ^{25}Mg, which are analogous to the GT transitions studied in the $^{25}\mathrm{Mg}(^3\mathrm{He}, t)$ reaction, can be obtained from γ-decay data [9] and also from (e, e') data [10].

The $M1$ γ-transition strength $B(M1)\downarrow$ (in units of μ_N^2) from an excited state to the ground state can be calculated using the measured lifetime (mean life) τ_m (in units of second), γ-ray branching ratio b_γ (in %) to the ground state, $E2$ and $M1$ mixing ratio δ and the γ-ray energy E_γ (in MeV). The relationship among them are given (see, *e.g.*, ref. [12]) by

$$B(M1)\downarrow = \frac{1}{\tau_\mathrm{m}} \frac{1}{E_\gamma^3} \frac{b_\gamma}{100} \frac{1}{1+\delta^2} \frac{1}{1.76 \times 10^{13}}. \qquad (2)$$

The $B(M1)\uparrow$ value that would be obtained in an (e, e')-type transition from the ground state (spin-value J_0) to the j-th excited state (spin-value J_j) is calculated by correcting for the $2J+1$ factors

$$B(M1)\uparrow = \frac{2J_j + 1}{2J_0 + 1} B(M1)\downarrow\,. \qquad (3)$$

The $B(M1)\uparrow$ values (to be expressed as $B(M1)$ for simplicity) to the excited states in ^{25}Mg, calculated using the available γ-decay data [9], are summarized in column 5 of table 1. Also the $B(M1)$ values from the $^{25}\mathrm{Mg}(e, e')$ reaction [10] are shown in column 8.

4 Discussion

4.1 Selection rules for spin and orbital operators

For the region up to $E_x \approx 6$ MeV in ^{25}Al, the GT states that satisfy the $\Delta J^\pi = 1^+$ selection rule are shown in figs. 1b), c), and in table 1. Among them only the ground, 1.613 MeV, and 6.122 MeV states are prominent, while all other states are strongly suppressed. The γ-decay and (e, e') data show a similar suppression of the analogous $M1$ excitations in ^{25}Mg, as seen from table 1. Significant $B(M1)$ strengths are seen only for the 1.612 MeV and 5.747 MeV states, which are the isobaric analog states of the 1.613 MeV and 6.122 MeV states in ^{25}Al, respectively.

As mentioned, the z projection K of the total spin J, is a good quantum number in a deformed nucleus with z-axis symmetry. We first examine the selection rules of the quantum number K and the asymptotic quantum numbers for the GT and $M1$ operators. The IV spin $(\sigma\tau)$ term is common in both of these operators. In addition, the $M1$ operator contains IV orbital $(\ell\tau)$, IS spin (σ), and IS orbital (ℓ) terms [11,12,27]. Therefore, we summarize the selection rules for the spin operator (σ) and also for the orbital operator (ℓ) on the basis of the study of ref. [28].

In intra-band transitions, quantum numbers specifying the intrinsic motion do not change. The matrix elements for the σ_z and ℓ_z operators are given by

$$\langle N\, n_z\, \Lambda\, K | \sigma_z | N\, n_z\, \Lambda\, K \rangle = 2\Sigma, \qquad (4)$$

and

$$\langle N\, n_z\, \Lambda\, K | \ell_z | N n_z\, \Lambda\, K \rangle = \Lambda, \qquad (5)$$

where Σ is the z component of the spin. In intra-band transitions, where K does not change, the transitions by the GT operator are allowed, and both orbital and spin contributions are expected in $M1$ transitions.

The inter-band transitions are caused by $\sigma_\pm$ and $\ell_\pm$ operators. By applying $\sigma_\pm$, we get

$$\sigma_\pm | N\, n_z\, \Lambda\, K \,\rangle \propto \delta(K, \Lambda \mp 1/2)\, | N\, n_z\, \Lambda\, K \pm 1 \rangle, \qquad (6)$$

showing that $\sigma_\pm$ operators cause transitions changing the asymptotic quantum number Σ, and thus K by one unit. It should be noted that the asymptotic quantum numbers n_z and Λ are not constants of the motion, but K is. Therefore, the selection rules $\Delta K = \pm 1$ are important results from eq. (6). In addition, it is expected that the transitions are much favored if the asymptotic quantum numbers n_z and Λ do not change in the transitions.

By applying ℓ_+, we get

$$\ell_+ | N\, n_z\, \Lambda\, K \,\rangle \propto | N\, n_z \pm 1\, \Lambda + 1\, K + 1 \rangle, \qquad (7)$$

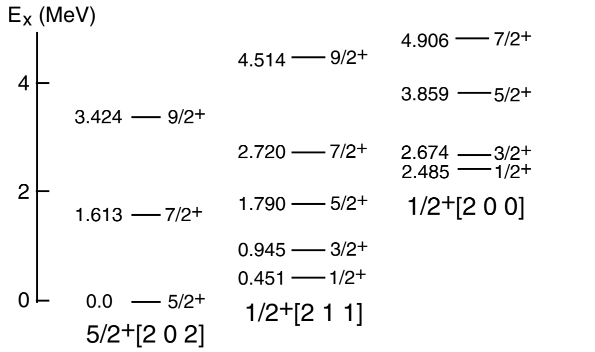

Fig. 2. Proposed band structure for the low-lying positive-parity states of ^{25}Al based on the Nilsson-orbit classification [3]. Each band is identified by the combination of quantum numbers $K^\pi[Nn_z\Lambda]$. Each state is denoted by the excitation energy (in MeV) and J^π values.

and by applying ℓ_-, we get

$$\ell_-|N\,n_z\,\Lambda\,K\,\rangle \propto |N\,n_z\!\pm\!1\,\Lambda\!-\!1\,K\!-\!1\rangle, \qquad (8)$$

where the relationships $0 \leq n_z \pm 1 \leq N$ and $0 \leq \Lambda \pm 1 \leq N$ should hold. Again the $\Delta K = \pm 1$ selection rules should be fulfilled. In addition, the transitions in which the asymptotic quantum numbers n_z and Λ change by one unit are favored.

4.2 Suppression of GT and M1 transitions

Our interest is to find out whether these unusually weak GT and $M1$ transitions in the $A = 25$ system can be explained on the basis of the selection rules. Due to the isospin symmetry nature of $T_z = \pm 1/2$ nuclei, similar structures of rotational bands are expected for the mirror nuclei ^{25}Mg and ^{25}Al, as discussed above. Their low-lying states form rotational bands based on Nilsson orbits (mainly consisting of $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ wave functions) of a neutron and a proton, respectively. On the basis of various experimental data [6,7,9], a band structure for ^{25}Al shown in fig. 2 and a very similar one for ^{25}Mg are proposed [3,6,7].

In fig. 2, we see that the transition from the $J^\pi = 5/2^+$ ground state to the 1.613 MeV, $7/2^+$ state is an intra-band transition, where the quantum numbers $K^\pi[Nn_z\Lambda]$ of $5/2^+[202]$ are assigned to the band. The GT transition to the 1.613 MeV state, therefore, is allowed both from the selection rule given by eq. (4) for the rotational band and the $\Delta J^\pi = 1^+$ rule for the state.

With our resolution of $\Delta E = 35$ keV, many weakly excited ^{25}Al states could be identified in the $E_x < 6$ MeV region. Among the weakly excited states observed in this region, the states at 0.945 MeV, 1.790 MeV, 2.674 MeV, 2.720 MeV, 3.859 MeV, 4.196 MeV, 4.583 MeV, 4.906 MeV, and 5.808 MeV have J^π values of either $3/2^+$, $5/2^+$, or $7/2^+$ (see fig. 1c)). These states are allowed in terms of the $\Delta J^\pi = 1^+$ selection rule. In such cases, the selections by the K quantum number should be examined.

As shown in fig. 2, the 0.945 MeV, $3/2^+$, 1.790 MeV, $5/2^+$, and 2.720 MeV, $7/2^+$ states are members of the rotational band $1/2^+[211]$. The transitions from the $J^\pi = 5/2^+$ ground state of the $5/2^+[202]$ band to the members of the $1/2^+[211]$ band require a change of the K quantum number by 2 units. These transitions are not allowed by the $\sigma_\pm$ operators, as seen from eq. (6), and thus GT transitions are not allowed. The same is true for the transitions to the 2.674 MeV, $3/2^+$, 3.859 MeV, $5/2^+$, and 4.906 MeV, $7/2^+$ states that are members of the $1/2^+[200]$ deformed band.

No simple deformed band structure with a single–quasi-particle configuration is assigned [3,6,7] to the 4.196 MeV, $3/2^+$, 4.583 MeV, $5/2^+$, and 5.808 MeV, $5/2^+$ states, or to any state above the $E_x = 6$ MeV region.

The above argument of the K^π selection rules for the σ operators is also true for the ℓ operators. As seen from eq. (5) and eqs. (7) and (8), exactly the same $\Delta K = 0$ and $\Delta K = \pm 1$ selection rules work for the inter-band and intra-band transitions, respectively. Therefore, the $M1$ transitions analogous to those GT transitions discussed above should behave the same. The fact that the analog $M1$ states are not observed or have very small $B(M1)$ values (see table 1) is well understood.

Each Nilsson orbit specified by the asymptotic quantum numbers is filled with two nucleons. Therefore, the increase of mass number A by 2 will change the ground-state configuration. The ground states of the $A = 23$ mirror nuclei ^{23}Na and ^{23}Mg are specified by the quantum numbers $3/2^+[211]$ (see fig. 1a)), while those of the $A = 25$ mirror nuclei ^{25}Mg and ^{25}Al, as we have seen, are specified by $5/2^+[202]$. Therefore, the transitions from the ground states to the states of the common $1/2^+[211]$ band have different natures of $\Delta K = 1$ and 2 in the $A = 23$ and $A = 25$ systems, respectively. Those transitions that were allowed in the $A = 23$ system are not anymore allowed in the $A = 25$ system.

Another interesting feature that became apparent from the comparison of these $A = 23$ and 25 systems is that at the deformation $\delta \approx 0.4$–0.5 the K selection rules are superior to the selection rules of asymptotic quantum numbers that would work first for very large axially symmetric quadrupole deformation. One can point out that transitions from the ^{23}Na ground state of the $3/2^+[211]$ band to the 4.357 MeV, $1/2^+$ and 5.291 MeV, $5/2^+$ states of the $1/2^+[220]$ band in ^{23}Mg are, in principle, not allowed by the $\sigma\tau$ operator due to the $\Delta n_z = 1$ and $\Delta\Lambda = 1$ nature of these transitions. They, however, are rather strongly excited, as seen in fig. 1a), because these transitions are allowed in terms of the K selection rule. Similarly, the strong transition to the 5.658 MeV, $5/2^+$ state of the $5/2^+[202]$ band is also K-selection allowed, although it is not allowed by the n_z and Λ selections.

4.3 Orbital and isoscalar contributions in M1 transitions

Both the GT and $M1$ operators have the same IV spin term ($\sigma\tau$), while the $M1$ operator additionally contains IV orbital ($\ell\tau$), IS spin (σ), and IS orbital (ℓ) terms. These additional terms can interfere either constructively or destructively with the IV spin term. Under the assumption that isospin T is a good quantum number, such contributions can be studied by comparing the strengths of analogous $M1$ and GT transitions. For $T_z = \pm 1/2$ mirror nuclei, the contributions of these terms were evaluated separately by making a combined analysis of the strengths of analogous $M1$ and GT transitions, as discussed in detail in refs. [8,27].

If the $\sigma\tau$ term, present in both GT and $M1$ transitions, is the main term, then there is a simple relationship between $B(M1)$ and $B(\mathrm{GT})$ values of analogous $M1$ and GT transitions [27]. If these transitions are between $T = 1/2$ states, we get

$$B(M1) \approx \frac{3}{8\pi}(g_s^{\mathrm{IV}})^2 \mu_N^2 \frac{1}{2} R_{\mathrm{MEC}} B(\mathrm{GT}) \quad (9)$$

$$= 2.644 \mu_N^2 \frac{1}{2} R_{\mathrm{MEC}} B(\mathrm{GT}), \quad (10)$$

where R_{MEC} represents the difference of reduction factors of the $\sigma\tau$ terms in τ_0-type $M1$ and $\tau_\pm$-type GT transitions due to the different contributions of meson exchange currents (MEC) [29–31]. The most probable value $R_{\mathrm{MEC}} = 1.25$ is deduced for nuclei in the middle of the *sd* shell [27]. From eq. (10), we find that by renormalizing $B(M1)$ as

$$B^R(M1) = \frac{2}{2.644\mu_N^2} B(M1), \quad (11)$$

the $M1$ transition strengths can be compared directly with the GT transition strengths $B(\mathrm{GT})$. The interference of IS and IV orbital terms with the IV spin term in an $M1$ transition can be evaluated by the ratio

$$R_{\mathrm{ISO}} = \frac{1}{R_{\mathrm{MEC}}} \frac{B^R(M1)}{B(\mathrm{GT})}, \quad (12)$$

where $R_{\mathrm{ISO}} > 1$ usually means that the IS term and/or the IV orbital term make a constructive contribution to the IV spin term, while $R_{\mathrm{ISO}} < 1$ means a destructive contribution. As discussed in ref. [8], the contribution of the IS term is usually small in deformed nuclei. Therefore, it is expected that the deviation of R_{ISO} from unity shows a contribution of the IV orbital term in an $M1$ transition.

The $B^R(M1)$ and R_{ISO} values are derived for the strong $M1$ transitions from the ground state of ^{25}Mg to the 1.612 MeV and 5.747 MeV states using the γ-decay and (e,e') $B(M1)$ values and the $B(\mathrm{GT})$ values of the analogous GT transitions listed in table 1. The R_{ISO} values for the 1.612 MeV state from the γ-decay and (e,e') data are both much larger than unity showing that the $M1$ transition strength is enhanced by the constructive interference of the spin and orbital contributions. It should be noted that this is an intra-band transition, and both spin and orbital contributions are allowed, as seen from eqs. (4) and (5). Since the concerned single-particle orbit $5/2^+[202]$ originates from a so-called high-j ($= l + 1/2$) type $d_{5/2}$ orbit in the spherical potential, Λ and Σ have the same sign, and thus the orbital and the spin contributions are constructive. On the other hand, the R_{ISO} values for the 5.747 MeV state in table 1 are both smaller than unity, suggesting the different origin of this state.

Using the γ-decay data [9] and following the procedure described in sect. 3.2, a $B(M1)$ value of 1.25(27) was calculated for the transition from the ground state of ^{25}Al to the 1.613 MeV excited state, the isobaric analog state of the 1.612 MeV state in ^{25}Mg. This value is more or less in agreement with the $B(M1)$ values of the analogous transition in ^{25}Mg derived in two different experiments, *i.e.*, $B(M1) = 0.83(12)$ from the γ-decay data and 1.2(3) from the (e,e') data (see table 1). As discussed in ref. [27], very similar $B(M1)$ values of analogous $M1$ transitions in $T_z = \pm 1/2$ nuclei show that the IS contribution is small. This small contribution of the IS terms can be explained by the cancellation of the IS gyromagnetic factors g_s and g_ℓ by the gyromagnetic factor g_R of the rotating core [3,8].

5 Summary

The GT transitions from the $J^\pi = 5/2^+$ ground state of ^{25}Mg to the excited states of ^{25}Al were studied by using the $(^3\mathrm{He},t)$ reaction at 140 MeV/nucleon and at 0° with a high resolution of 35 keV. If the normal $\Delta J^\pi = 1^+$ selection rule is applied, states in ^{25}Al with the J^π values of $3/2^+, 5/2^+$, and $7/2^+$ can be excited by the $\sigma\tau$-type GT operator. However, in the low-lying region $E_x < 6$ MeV, only the transitions to the ground and 1.613 MeV states were prominent, while several other transitions that could be allowed by the J^π selection rule were strongly suppressed.

In the middle of the *sd* shell, nuclei are largely deformed, and low-lying states of the mirror nuclei ^{25}Mg and ^{25}Al are well described in terms of the particle-rotor model assuming that an odd nucleon is in various Nilsson orbits labeled by quantum numbers $K^\pi[Nn_z\Lambda]$. According to the K selection rules for the GT operator, either intra-band transitions or inter-band transitions with $K \to K \pm 1$ are allowed.

It was found that the GT transitions from the ground state of ^{25}Mg to the ground and 1.613 MeV states in ^{25}Al are intra-band transitions, *i.e.*, $\Delta K = 0$. On the other hand other, several weak transitions are inter-band transitions with $\Delta K = 2$. In this way, the suppression of the "J^π-allowed transitions" is well described by the K selection rules for the GT operator. The suppression of $M1$ transitions in ^{25}Mg from the ground state to the analog states of the GT states can also be explained by the K selection rules for the $M1$ operator. It should be noted that such "well-working K selection rules" suggest a good axially symmetric shape of the $A = 25$ mirror nuclei.

The orbital contributions for the $M1$ transitions in ^{25}Mg were also studied. It was found that the intra-band $M1$ transition to the 1.612 MeV state was enhanced by the constructive contributions of spin and orbital terms of the $M1$ operator. On the other hand, a destructive contribution was found for the transition to the 5.747 MeV state.

The small $B(\mathrm{GT})$ values of less than 0.04 listed in table 1 may be less reliable [24]. It is, however, noted that although the reliable B(GT) values are obtained from a β-decay study of ^{25}Al, such study cannot give GT strengths of the states lying higher than the decay Q value ($Q_{\mathrm{EC}} = 4.277$ MeV). In addition, even if the corresponding B(GT) values are considerable, it may not be easy to detect GT strengths with smaller decay energies because of the smaller phase space factors inherent in the decay. In contrast, we had no such difficulty in the study using the (^{3}He, t) reaction. Among the small B(GT) values, only the value of 0.003 ± 0.001 for the transition to the 0.945 MeV state in ^{25}Al (see table 1) can be compared with the value of 0.0024 ± 0.0003 for the analogous β-decay from the ^{25}Al ground state to the 0.975 MeV state in ^{25}Mg [9]. The small B(GT) value derived by using our standard procedure from the (^{3}He, t) measurement assuming the proportionality (eq. (1)) is indeed within the error of the value from the β-decay. Therefore, from our present data and analysis, it is safely concluded that below $E_x = 6$ MeV in ^{25}Al there are no states, other than the ground and 1.613 MeV states, that carry a considerable amount of B(GT) strength.

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References

1. F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992) and references therein.
2. J. Rapaport, E. Sugarbaker, Annu. Rev. Nucl. Part. Sci. **44**, 109 (1994).
3. A. Bohr, B. Mottelson, *Nuclear Structure II* (Benjamin, New York, 1975) and references therein.
4. P. Raghavan, At. Data Nucl. Data Tables **42**, 189 (1989).
5. M. Guttormsen, T. Pedersen, J. Rekstad, T. Engeland, E. Osnes, F. Ingebretsen, Nucl. Phys. A **338**, 141 (1980).
6. A.E. Litherland, H. McManus, E.B. Paul, D.A. Bromley, H.E. Gove, Can. J. Phys. **36**, 378 (1958).
7. F. Heidinger, P. Betz, W. Brendler, F. Glatz, A. Hoffmann, H. Röpke, B.H. Wildenthal, Z. Phys. A **338**, 23 (1991).
8. Y. Fujita, Y. Shimbara, I. Hamamoto, T. Adachi, G.P.A. Berg, H. Fujimura, H. Fujita, J. Görres, K. Hara, K. Hatanaka, J. Kamiya, T. Kawabata, Y. Kitamura, Y. Shimizu, M. Uchida, H.P. Yoshida, M. Yoshifuku, M. Yosoi, Phys. Rev. C **66**, 044313 (2002).
9. P.M. Endt, Nucl. Phys. A **521**, 1 (1990); **633**, 1 (1998) and references therein.
10. L.W. Fagg, W.L. Bendel, E.C. Jones jr., H.F. Kaiser, T.F. Godlove, Phys. Rev. **187**, 1384 (1969).
11. E.K. Warburton, J. Weneser, *Isospin in Nuclear Physics*, edited by D.H. Wilkinson (North-Holland, Amsterdam, 1969), Chapt. 5, and references therein.
12. H. Morinaga, T. Yamazaki, *In Beam Gamma-Ray Spectroscopy* (North-Holland, Amsterdam, 1976) and references therein.
13. W.G. Love, M.A. Franey, Phys. Rev. C **24**, 1073 (1981).
14. M. Fujiwara, H. Akimune, I. Daito, H. Fujimura, Y. Fujita, K. Hatanaka, H. Ikegami, I. Katayama, K. Nagayama, N. Matsuoka, S. Morinobu, T. Noro, M. Yoshimura, H. Sakaguchi, Y. Sakemi, A. Tamii, M. Yosoi, Nucl. Instrum. Methods Phys. Res. A **422**, 484 (1999).
15. T. Noro *et al.*, RCNP (Osaka University), Annual Report, 1991, p. 177.
16. Y. Fujita, K. Hatanaka, G.P.A. Berg, K. Hosono, N. Matsuoka, S. Morinobu, T. Noro, M. Sato, K. Tamura, H. Ueno, Nucl. Instrum. Methods Phys. Res. B **126**, 274 (1997) and references therein.
17. T. Wakasa, K. Hatanaka, Y. Fujita, G.P.A. Berg, H. Fujimura, H. Fujita, M. Itoh, J. Kamiya, T. Kawabata, K. Nagayama, T. Noro, H. Sakaguchi, Y. Shimbara, H. Takeda, K. Tamura, H. Ueno, M. Uchida, M. Uraki, M. Yosoi, Nucl. Instrum. Methods Phys. Res. A **482**, 79 (2002).
18. H. Fujita, Y. Fujita, G.P.A. Berg, A.D. Bacher, C.C. Foster, K. Hara, K. Hatanaka, T. Kawabata, T. Noro, H. Sakaguchi, Y. Shimbara, T. Shinada, E.J. Stephenson, H. Ueno, M. Yosoi, Nucl. Instrum. Methods Phys. Res. A **484**, 17 (2002).
19. Y. Fujita, H. Fujita, G.P.A. Berg, K. Harada, K. Hatanaka, T. Kawabata, T. Noro, H. Sakaguchi, T. Shinada, Y. Shimbara, T. Taki, H. Ueno, M. Yosoi, J. Mass Spectrom. Soc. Jpn. **48**, 306 (2000).
20. H. Fujita, G.P.A. Berg, Y. Fujita, K. Hatanaka, T. Noro, E.J. Stephenson, C.C. Foster, H. Sakaguchi, M. Itoh, T. Taki, K. Tamura, H. Ueno, Nucl. Instrum. Methods Phys. Res. A **469**, 55 (2001).
21. C.D. Goodman, C.A. Goulding, M.B. Greenfield, J. Rapaport, D.E. Bainum, C.C. Foster, W.G. Love, F. Petrovich, Phys. Rev. Lett. **44**, 1755 (1980).
22. T.N. Taddeucci, C.A. Goulding, T.A. Carey, R.C. Byrd, C.D. Goodman, C. Gaarde, J. Larsen, D. Horen, J. Rapaport, E. Sugarbaker, Nucl. Phys. A **469**, 125 (1987) and references therein.
23. W.G. Love, K. Nakayama, M.A. Franey, Phys. Rev. Lett. **59**, 1401 (1987).
24. Y. Fujita, H. Akimune, I. Daito, H. Fujimura, M. Fujiwara, M.N. Harakeh, T. Inomata, J. Jänecke, K. Katori, A. Tamii, M. Tanaka, H. Ueno, M. Yosoi, Phys. Rev. C **59**, 90 (1999).
25. Y. Fujita, Y. Shimbara, A.F. Lisetskiy, T. Adachi, G.P.A. Berg, P. von Brentano, H. Fujimura, H. Fujita, K. Hatanaka, J. Kamiya, T. Kawabata, H. Nakada, K. Nakanishi, Y. Shimizu, M. Uchida, M. Yosoi, Phys. Rev. C **67**, 064312 (2003).
26. DW81, a DWBA computer code by J.R. Comfort (1981) and updated version (1986), an extended version of DWBA70 by R. Schaeffer, J. Raynal (1970).
27. Y. Fujita, B.A. Brown, H. Ejiri, K. Katori, S. Mizutori, H. Ueno, Phys. Rev. C **62**, 044314 (2000) and references therein.
28. J.P. Boisson, R. Piepenbring, Nucl. Phys. A **168**, 385 (1971).
29. A. Arima, K. Shimizu, W. Bentz, *Advances in Nuclear Physics*, edited by J.W. Negele, E. Vogt, Vol. **18** (Plenum, New York, 1987) p. 1, and references therein.
30. I.S. Towner, Phys. Rep. **155**, 263 (1987).
31. A. Richter, A. Weiss, O. Häusser, B.A. Brown, Phys. Rev. Lett. **65**, 2519 (1990).